

KOREŠPONDENČNÝ MATEMATICKÝ SEMINÁR

MATIK

Číslo 5 – Ročník 32

matik.strom.sk





Ahojte!

dbYfS- YP<- ^-\ S- Y<CY-ebYfS- CzCeQ@^-\ SF, CzW<P ^-s zC zcY Uq^
 eb< sS>Wbq Pb eq<Pb@^-\ eqG q@-> C b^C@Pb ^-s <- W-, CW^b<>^b bW<A
 ^Us- ^G-@q- zC^C 4Y S- UzG\ ^ @q-PCUs qfi y- W^Cbz- U<G> qC C^S YP \ C
 eP^Ss Y^ bsbPi dq U C f-> \ ^bPb ~ Szb<^ <P ^-e-@bf- \ %S^bWeq qC C^ i
 , - S \ Sbf-^ fC@ <S.MATIK-

Ako bude

Tábor mladých matematikov

, Uzbzb Yzb \ C szqfS z @C eY <-4 f%as W\ -qz\ S- s-eCq fC@ <S S^<-
 y-4bc\ Y@ <P \ -zC -zSWfi | C s- zCS ^- ^Cbe-Wbf-zC^ eqLq \ >-4 f^C
 eb@^ \ -zSW - eqU \ ^ sebYb<^bs i
 y | | s- 4-@CW^ - cci QcDi -~L-sz- f; P- zCp-@S eqsb4-S, %dUrY^CU- U
 ~qC^ eqC4-@ <SP sS@ \ -Wbf - 4-@ <SP @q-P-Wbf ^- szqC^CU WYi Vb\ eYz^
 S^Hq \ <S> -W - UeqPYsbf- ^S<^ ^-L@C ^- ^- Cuszq^W | C^GP-f UsSeqPYs^C S
 ^- ebsY<^ <P f ~>YAb eb<Z \ Ssz Ucb4 C@C^ i yC \ Cs- ^- zC4-i

2% z daní

, UzCzb qW \ ~ f- Sqb<S f^bf | h <b sfbUSP @^ fCqU beqpsC ^ \ bL-Q
 ^S-<S> >- Ws\ C \ %f^S Wbq @W^<- - {hg
 dC^S<C< sW^ < | h f%o f- \ C^ - ebWp<S<-szS^ -W @bf sebuC^ <P s- WsYz \ S
 eq f-s fzY< <-sbeSbf>eb zb f^ ><C^%<- s - S <P>- WsYz%<- s szqC^S <P g
 ; P<C C f-s eqzb ebeqS>- 4%szC qb<S\ > <C^b\ sfbU 4Y WU- U f<@S Y^CU
 qb<S%<-sC@b\ - ebWU C- U<-@ \ ~yb\ ^- ~YSebqpeq f Yb^ - SP- WsYz- <P
 - ebeqS SSSP>- 4%sfbl~ zq W~ eb@ebqS^ - ~ @b4qfb ^ <W C^s^bs - eb\ bPYS
 z \ sWeS^C \ Y@ <P <S- fC@b\ <P ~@ <- 4GeC<-U <SP <Pb@z <Pzb -s^ <P sCQ
 \ S- <bf> Wbq z W<4b ~UzG dbqpeq f U<S \ >cb eq f-s <^-\ C^<-s szqC^S<>
 qb f-> @f qC C^S YP sC S- q->- f%<fC^S<S \ > Cz- Wb eb@ebq - Uf- qf fbU
 - eqseCU W \ %Y eY \ ~ zq fC^S f- Pb fb ^ Pb <-s-i
 dbzqC4^ S^Hq \ <S b zb \ >- W @ qb- | h >^ -L@Cz ^- szq^W ^-Pb <@q C^S
 <@q- C^S G szqb\ i s Ws Ww @q- C^S w\ eCq<C^z- w- q @S f-> b@ebfS C ^- ~4bQ
 fb ^ bz<-W%bP -@b\ ^- C Ueb@ebq% U \ -Sb\ ^- S^Hb2szqb\ i s W
 p- WU C F

Vzorové riešenia 1. série úloh letného semestra



beq fb f- Š[- b K4 q- d- b d- b fç W

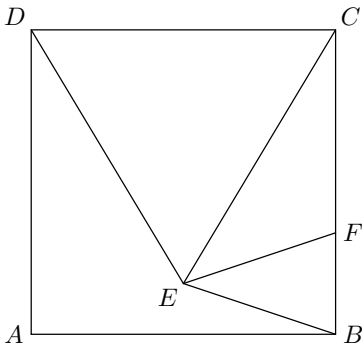
85 çC C

^- Uq USC çC C S=, Sx ç zçWf-

Zadanie

db @Pb\ fçççfb f^ zçs- Y^SW fzf- ç zfbç- ABCD C beš WeC E z W C zçU-PbY WDEC C çf^bszq ^^ i] - Pq ^CBC C eçF >eççb |EB| = |EF|i , W-C fçWš ~PY CEFm

Riešenie



a < ^- ç\ C sš @ W szq ^%o zfbç- ABCD ai yçU-PbY WDEC C çf^bszq ^^ - szq ^- CD C - U szq ^b- zfbç- > z WC f C WšçççPb szq ^%d - U @ W a - C P b f^ z b ç ^ - P Y %d - U fçWš 60° i yçU-PbY WEBC C çf^bç \ C ^ sb < - W @ b- EB > Vç C |EC| = |BC| = ai } PbY beçzç - W @ S z b P z b zçU-PbY W C ~ PbY ECB - C P b fçWš C |ç ECB| = |ç BCD| - |ç ECD| = 90° - 60° = 30° i Vç C C ç U ç P b Y ^ WEBC çf^bç \ C ^ - ~ P Y %BEC - EBC eççC P b < - W @ S ç f^ - W š fçWš > S P fçWš C $\frac{180^\circ - |\angle ECB|}{2} = \frac{180^\circ - 30^\circ}{2} = \frac{150^\circ}{2} = 75^\circ$

Vç C |EB| = |EF| > zçU-PbY W B F E C çf^bç \ C ^ s ç \ C ^ \ S E B - E F i } fç C > C |ç EBC| = 75° - ~ P b Y E B F C z C š z - W š E B C i , ç f^ b ç \ C ^ b \ zçU-PbY W s ~ P Y %eçç - W @ S < P b @ > ç S C - U |ç BFE| = 75° i , C Wš ~ P Y B E F C |ç BEF| = 180° - |ç EBC| - |ç BFE| = 180° - 75° - 75° = 30° i , C Wš ~ P Y C E F C ç @ ç b \ fçWš z ~ P Y f B E C - B E F > ç S C |ç C E F| = |ç B E C| - |ç B E F| = 75° - 30° = 45° > ç b C fçWš > W b ç s \ C e b z ç C b f š < š z š i

Komentár

YP- eç fç ç S ~ < f-s ^C4bY ^-çç^ i] b fš < Cq < f-s szq zšš4b@%- ^C@bW- ^šC s W z b ç ^ b s z > W b ç s < C f % C q U b ç š f š @ > - Y C C e b z ç C ^ f ç C C ~ W - > e ç ç b e Y z š i] - e q W @ e ç ç b C zçU-PbY WEBC çf^bç \ C ^ i , ^š W b ç < P f - S P ç C C S < P z C s - W f s Y @ W @ b e ç < b f - š z - W C z ç s š f š š C zçU-PbY W š E B C - B F E s s S e b @ b ^ > Vç C s b 4 ç f^ b ç \ C ^ - \ - U < P b @ - P b Y e ç ç - W @ S ç b 4 b y z ç < C W \ C C L ^ z se sb 4 ç C C S Y P % a 4 U f š š s - f - W ^ b P b ç C š C b f > W b ç ç C š š Y P - Y ^ ç b f - ^ - b @ C q ^ \ fçWš z - P Y > ç b e ç Y P - P z b z b z @ ^ ç z - ç > Vç C ç f ^ b f ^ š C ^ š C C e ç ç s se sb 4 - W š @ b e ç < b f - W f s Y @ W i a z \ s f ç ç - U H W > C f ç S - z W < P z ç C C s @ b z - Y W P % a \ ~ f s Y @ W > Y S < C ~ s - b @ z b P b s e ç f ^ C P b b e - q z s - e b f i



beq fbf YGSfe ; sb^W - [S SO- ^~s
^~Wq USC qC C^S=r- \ ~CYa s-sW

Zadanie

} ^GvWp^ Pb @SfS W \ - Y eq@ 4SW- zqas- szbYf bx^~c^ <P eqfb@C^ \ S
çsY \ S| - {ci dq fC@f- szb%e- zqSS@b s-Y^SWi, 4%eGpb^~YeqfS^fC^z çC^SzS
Wbq @f- zb s >eb~ f-YzqSW] - @fCq~P s-Y^SW 4bY z-4- W s çsY\ >Wbq
^C4bY @CS^ çsY \ S szbYf <b s-Y^SW>- Y çsY \ S f CzW~P bsz-z^ <P szbYf
~bi a Wq\ zbP>çsY szbYf f s-Y^SW ^-sY@bf- Yeb sC4G Vzbq @fCçsY zb 4bYm

Riešenie

, Ws- ^CJW n f%vWz~C \ C@SçsY \ S szbYf s-Y^SW> C^Sz~C çsY t fçsY ^-
z-4- Wg @CS^ bsz-z^ \ S çsY \ S < @^ Pb çx s-P~ bWq\ n - s-sC@^ <P eqf
ç@C^ <P çsC^~ Y^@CS^ ni Š zbP eY%SC> C f eqfbçsCYb\ çx W@C \ ~s
\ - n^S Wbq eqfbçsY fS <Wz^C f CzWbsz-z^ çsYi, çx W@C t s^S Wbq p
< eqfbçS^S Cbf n zbs \ ~s f%vWz^ \ C Uq~^C f n>- Ysz~Y-seb zb Wb~W W
f Wbç\ W fCW bsz-z^ <P çsCYbWq\ s-sC@^ <P>eqfb\ S^~sb4W@p s-Y s b@
n-seb b p>^S b ci
VC4%an 4bY s çS^b\ ^CJW<P @fb~P ^G @CS^ <P çsCYq <^%P b@c>W @ t
@CS^ z \ Sb @fb\ SçsY \ SfWbq >- W @CS^C n q <^Cb@c>s fç SC^C c>- Y
\ C^ SC^C n-1>çS C@CS tg 4%4bY @CS^ -USP s çS^b\ ni yb U sebp Y4b
n <b <-@^S ^C@Y t>zC@ n \ ~s 4%o\ b<^S^b- eqfbçsYi
] - W W P - @^ çsY ^-sY@-U eb sC4G~W^b <^S P U e-q^G yb \ C4%o\ b<^S
^b~ U@S Pb eqfbçsY>- zb |i Š-çfC zb \ ~s 4%o^- U%oS < z <Pz \ b<^>-4%o
s- f^C U@S eqfbçS^S f%vWz^f-YfS <^sb4^C^C f S^ <P çsY~Pi
, <-@^b\ çx s-P~ U^~ U%ob~ \ b<^S^b- @fbUWçsY cv>Wbq \ ~s 4%os ç-s b-
W @ Pb qC C^S i a fCq\ Cb4Cs-sC@^ çsY> Wbq <P S^~ cu U\ b<^S^b- eqfbçsYi
-PW b fCq\ G CS^ ^~sb4bWcu \ C@SçsY \ S^C \ G-z- WCz-zb @fbIS- sWzç^C
f%b f~U

Komentár

, sY@Wb- eb@ qS^~U \ ^bP \ i? bW^<- U@C^o 4b@bf <P qC C^ 4bY ^C \ Yi
S >^~ç^~çs <^SP ^C4bY z-W> W 4%a\ CsS\ bP^Seqf i, C- < f-s-s çx Pb@Y
sW - f CzW~P {Cç sCYfçSeCW fC f CzW~P |_ @fbUg- eqf W @\ < fç S^%
f%e-s @fb@eqçb fPb@ çsY^~ z-4- W^C\ CeqC@bz^q^ \ b^bs C^Szbf >
çb Szb f%o @bf-Y <4%bç^ ç4bz-i y-Wzb qC C^S 4%eq f@Ceb4^C ^C-seCS
-W4%4bY szbYf fS < çx Y S çb SY^ e-qzS <çsCY^~ eqfbçS^S C C U^C Pq~^~
szq z ç-s-> Y^~U^~ U%o ç S \ b<^S^~ @fbUW@Pb^Czqf+i] - ebW^ f - Wf P - @b\
^~ <-@^ Pb@bz%o UqC C^S eqP -@f U <C \ b^bszSebfç S^C < sW^SeY ebçz%o
Wç C4bY^~eqfCW f CzW \ ~ WqCW^ i

{ beq fb f SX-U- [S b z b f - [- q s z c f v b .
^ - W q USC q C C S = B f W V q U S b f -

Zadanie

r - \ beb c z H ^ L - U e q s ^ C - W W W Y c W i O b s z s ^ s W < P < C Y ^ - s - \ beb c z s z ^ S W W z p U S q ^ < P e q f @ C ^ < P c s C Y] - e q f e b W s @ b s z - Y f s Y @ b W || J i d q W ^ z p Y s z z b c s Y < ^ b f - - f % o b \ ~ | c _ D i d q z b s z z b c s Y C z C q < - z C q < @ b s z - Y s c z || C i d S z C e q f e b c z z f ^ c s Y < - @ Y z b z S f @ % ^ C s e q f ^ C Y 4 b e f W @ \ e b W s C ^ C s z Y c S Y ^ S W b q < U P b < S C q @ b s z - z b c ^ C s S Y b - @ b s - \ beb c z - z W - @ Y f @ % ^ - \ S z b z p U S q ^ P b c s Y c s Y @ f b U S q ^ i S @ C y - Y S C < P % % e f s z z f ^ ~ ^ C e q f S Y - s - \ beb c z z W S z C ^ z b W z H ^ L b f - Y 4 C < P % ^ G , W U C s e q f ^ % s c z P b s z s W P b c s C m

Rieszenie

d c f e b c 4 ^ < P Y P < P U C @ b 4 q f % o S z c f W ^ G ^ ^ C c s Y b s s ^ - e s - e b \ b < b - U P b < S C q , W s - z @ U @ - b { q S q ^ c s Y b ^ - e C C P b - W x y z > e q f b \ f S C G > C x > y - U z s < S f % z C e q f @ C ^ c s Y f y - z \ ~ 4 % o - U C G ^ C S C - W c C e d c z W b < - e s ^ b \ c s C f S @ > C b 4 s - P - U C x z b f S W y @ C s z b W - z U @ b z C W z - W C P b \ C C e q f e s - - U ^ -

$$\overline{xyz} = 100x + 10y + z.$$

r c z f z W < P c s C y W q e S z P b s S b x ^ - c \ C n i d S z C c s Y b s S b x ^ - c \ C a b c i O b s z s ^ Q s W e f e b c z Y W h q < a b > q < a c - q < b c > - Y C ^ C f S C G - W @ % e f e b c z Y W b q i V G y C c s Y a c - b c \ - U c f ^ - W < S q ^ - \ S z z C U @ b z S W z W U c s Y n + a c - n + b c 4 - @ \ - c f ^ - W < S q ^ - \ S z z C U @ b z S W y b s c s Y || J - || C i y \ e - @ \ < f % ^ < b s c z b f > W b q P b s z s W @ b s z - Y c S C | c _ D 4 - @ C n + a b i V G y C c s Y a c - a b \ - U c f ^ - W < S q ^ - \ S z z C @ C s z b W z - W c p @ C Y c s C Y n + a c - n + a b 4 - @ C =

$$(n + 10a + c) (n + 10a + b) = n + 10a + c \quad n \quad 10a - b = c - b,$$

c S C S P c p @ C Y 4 - @ C U @ ^ b - S q ^ i T C @ S U @ ^ b - S q ^ c p @ C Y U C - Y c p @ C Y c s C Y | C E - | c _ D i d q z b c s Y || C E 4 - @ C n + a c - c s Y || J e b z b \ \ ~ s 4 % o n + b c i S c p @ C Y (n + b c) (n + a c) = 2224 2204 f % Y f > C b a = 2i S c p @ C Y (n + a c) (n + a b) = 2204 2198 f % Y f > C c - b = 6i y S z b c f ^ S C s z \ C - @ b s z ^ C \ C b a + c b = 8 > c S C c a = 8i V G y C c s Y a b c U C z p U S q ^ c s Y - a > b > c s < S f % z - W a > 1 - e b z b \ b s z - f U @ S ^ \ b ^ b s z - W > C c - a = 8 > - z b a = 1 - c = 9i S z b P b f % Y f > C b = 3i d S z C e q f e b c z z f ^ c s Y 4 b Y z @ c \ _ i d b z b \ < C W f s c z \ S z \ C b @ e b c z ^ \ ^ C e q f ^ C P b c s Y b @ s c z - > W W b q \ ~ e - z q > - e f e b c z ^ \ s e q f ^ C P b i r e q f ^ % s c z 4 b Y { | J f 224 39 + 139 = 2324 g

Komentár

, °ç S- < f-s \ - Y @b4q \ \%šC'W>Wčy szC YBP~ qčSS- f - Wç- szb s- f- \ sz-Q
 f- Y ^C'W-@q4^--P%4 i] -eqWf@szC<-4-@š< f-S ^SCWbq eqe-@%o Y4b ^SCb
 ^C@bsz- zbc^Cebe s- šI , b f Cb4C^bszS f - W YBP- @be-@Y @b4qI , Ws- eqz- WUzb
 YPC čx Pb@'Cz ebsz-ebf- f fS < çS \ ^C'Uğ s'šzC \ -zšW\ sW - ^ \ \ b ^bsz >
 \ ~s zCš- ~šzS > CszCSP ^-bx- Uf%šW - šYf C'W%š-ç Uzb Y' eqU@C^SC^SCW W\ S
 \ b ^bs - \ š dčzb b@ebq ç- \ CP-@ qč C^SC>Wbq ^Cf%o f- sW - ^šI

J beq fb f šV ~4b G q4~Y - ? - ^b a ^@
 ^- Wq USC qč C^SC=X-W- T- <Wš

63 qč C'

Zadanie

] - eG-S Yb\ szbYCW>Wbq PbszS sW f%šCsbY<b <-@C'ç- szSW<P%šC>4bY ^-
 ebx-@ ~\ CC'WPb szf-q' C^S ^SCW@ UCWšP bf-zW%o- eqbszq@ @šfbcS'%of%qç- ^-
 \ qč W 8 x 9 ebYčbW, U Pbx-e-@'b\ čp~ febYčWš A1g YC Pq < W\ C i dbP%š
 4-Učs- < ebYčW ^- ebYčWš] - eY<PC'UzšC çšCq' %sz e><G Wbq s- ^C@-ebP%šbf-
 f- ^SeqšWšC^ \ g ? f- U Pq-çSs- szqč@ U f - Pb<P W\ C b\ b ~4bfb ^ ebčCz
 ebYčbW^- f <Pb@- Y4b sCfCq , - <b\ Uč zC^>Wš @bsz- ^C W\ C šfbU\ - Pb\
 @b <SC- febYčWš H9g p b Pb@šC- b@ fb@šC-çSeq^SCWbq Pb < Pq-čbf Cš.Sz-Uč
 f - < ^-szq z LS i , W^b>ebe zCU i f , - < ^-szq z LS Uč ^-fb@>- Wš \ -Pq-ç Pq >
 -4%f @%f%Bq Y^C'P zC^ @q-P Pq- WšWš fCWg

	1	2	3	4	5	6	7	8	9
H									
G									
F									
E									
D									
C									
B									
A									

Riešenie

YBP~ \ ~s \ C<-ç- qčC S b@W^<->zC@ <šzS>< Wbq <P ebYčbWUč ^- UčC^ -P
 ~ šz-f Pq i] - WšWš <Sčbf ebYčWš Uč H9 - fšC\ C s- ebP%šbf- š4- čf^b ^-
 f <Pb@- čf^b ^- sCfCq z Wšzčb f%Bq f U<C ebYčW s = <CY qš @bWH - <CY
 sz eG 9i p- Y \ WšWš 4-@C^UčC^SC ebYčW>< Wbq Pb s- ~ ~qšC @bsz- ^C\ C
 ^- f%Bq f U<C ebYčWi , š@ \ G C UčS z Wzb ebYčWš Uč G8i š z Pb s- zbzS Pq-ç
 ^- -P~ \ ~s ebP^ ^- sCfCq- Y4b ^- f <Pb@- f b4b<P eqe-@<P s- @bsz- ^C

\wedge - f%Rq-f U<C eBYçWbi | C C ebfC@ > C eBYçWb G8 UC eqPq-f U<C eBYçWb>
 eqçz C ^C-P Pb-SJW -P s- < ^Pb ~ç4 > -P-U <SPq-ç sWb^ç ^- f%Rq-f U<b\
 eBYçWi , %Rq-f U<C eBYçW s zC@ zC> < Wbq <P s- fS\ C U@^ \ -Pb\ @bsz-
 ^- eqPq-f U<C eBYçWb- Y4b @b <S- i ; P<C G- 4%δ— s eCq<-ç ^- YsfbUC -P%δ-
 eqPq-f U<S-P eBYçW<P>- \ %δ- ^SP <P<C CsfbUC -P%Wb^çSi p- YSeqPq-f U<C
 eBYçWb UC F7>eqçz C- Ws- < ^Pb eqzPq-ç ebP^C Pb<SW\ > \ %s- < zbPb eBYçW
 @bsz- ^C C4-ÿ @b <S- >- Y4b ^- eqPq-f U<C eBYçWbi
 y-W\ zb se sb4b\ fS\ Cb@P- Y f CzW%eqPq-f U<C eBYçWi ySs =C6>eqçz C
 s- < ^Pb fS\ C @bsz- Y^ ^- C7Q9 - zCs f CzW%of%Rq-f U<C B5>eqçz C
 s- < ^Pb fS\ C @bsz- Y^ ^- B6Q9>C5 - zCs f CzW%of%Rq-f U<C A4>eqçz C
 s- < ^Pb fS\ C @bsz- Y^ ^- A5Q19>B4>C4 - zCs f CzW%of%Rq-f U<C F3>
 eqçz C s- < ^Pb fS\ C @bsz- Y^ ^- G3Q13 - zCs f CzW%of%Rq-f U<C E2>
 eqçz C s- < ^Pb fS\ C @bsz- Y^ ^- F2Q12>E3 - zCs f CzW%of%Rq-f U<C- D1>
 eqçz C s- < ^Pb fS\ C @bsz- Y^ ^- E1Q11>D2>D3 - zCs f CzW%of%Rq-f U<G
 , S\ G C ^- Wb^S ^-Pb -P- <P<C C sz- ^- eqPq-f U<b\ eBYçW - <-çfC
 <-ç ^- C ^- A1i , Ws- <P<C C @bsz- @b f Pb@Cueb <S> \ ~s \ C s -Wb eqf
 - ebP^ s ^- D1 -Y4b A4 - ebzb\ ~ sz-YC -P- S4- z-W-4%δ\ C s @bsz- Y
 ^- eqPq-f U<C eBYçWb>- Y4b @b <S- >- Wzb bWYbszS ebfb ~Ui dçz f PCq^—
 sqz z LS C Sz-UC eqç eqf Pb Pq-ç i

Komentár

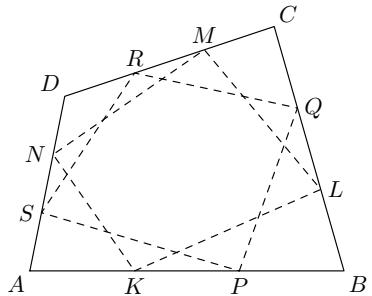
YP~ @b4ç f%çC S b fC- < f-s> f -Wf \ ^bP <P çC C'S <P s- f%W%S @ç4^ <P%Q
 4çW%ol ^bP seqf^C ~çS f%Rq-f U<C- eqPq-f U<C eBYçW>- Y^ ^b@ fb@S
 eqçb ^- ^SP Pq-ç < sW f Pb@-w^çf Pb@->4G zbPzb b@ fb@^C'S UC çC C'S ^C Q
 eY >- zC@ s \ C<- ^C bPS~@CS eY ebcçz 4b@çfi p-Y \ ^C@Wb^Y \ çC C^ \
 4bYb>Wÿ zC f%ç s-Y <-q @b\ f CzW%WpW%o ebfC@ Y- C4G bP-@ ^- s eCçfC
 WpW%o-çSç f%Rq-zç 3bÿ zC@ S4- be s ^—sqz z LS >^Sç f -Wse sb4>Wbq \ zC
 W^CU@bseCS- Y4b UC @ fb@^C'S i

I beq fbf- Y [-z [-sq^ - V çsz ^ [SY^bf—
 ^- Wç USC çC C'Sç-V-zW G ç4-Yf—

47 çC C'

Zadanie

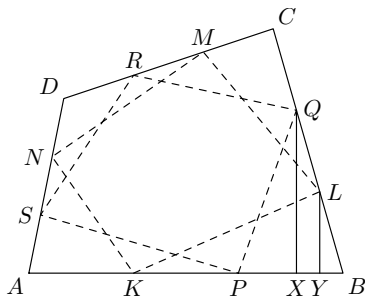
[Ssz^bs \ -zf-qWb^fç^ Pb zfbq-PbY W
 ABCD fzb<^-\ C^→ Cf CzW%UPb f^ zbq^
 ~P%o\ -U \ C^CU- Wb 180°g V- @—szC^—
 \ Ssz^bszUC @fb\ - szbY Sçç @CC^—^— zçf
 çf^—Wb @P sCW%o Wb ^- b4q- W fszbY%ç
 K>P>L>Q>M>R>N>Sg ? bW-zç Cb4YszS
 KLMN - PQRS \ -U <Pb@^ b4s Pi



Riešenie

a 4s-P zfbq-PbY W KLMN fS\ C f^Ad@cS -U- Wb b4s-P <CYPb zfbq-PbY W ABCD \ ^-s b4s-P%zqU-PbY Wbf KBL> LCM>MDN - NAKi pbf^- Wb b4s-P zfbq-PbY W PQRS fS\ C f^Ad@cS - Wb b4s-P zfbq-PbY W ABCD \ ^-s b4s-P%o zqU-PbY Wbf PBQ>QCR>RDS - SAPI, WzC@ ~W-C G C b4s-P%z <Pzb zqU-PbY Wbf s- cPj^-U> ~W-C C-U C s- cPj^-U b4s-P%o zfbq-PbY Wbf KLMN - PQRSi

dbx dS C s- ^- @fbIS~ zqU-PbY Wbf f UC^b\ cP~ zfbq-PbY W ABCD fzC@ SP sebYc^ f q-PbY UC f q-PbY zfbq-PbY W ABCD g^- eqW@ ^- zqU-PbY W%o KBL - PBQi ? b^- eq~ @beY\ CSP f W%o 4b@bf Q - L - SP e^o Z%b^ - c\ CX - Y = ? W f W%o LY sSb^- c\ Cv - @ W szq ^%o PB sSb^- c\ Cai dbx dS C s- ^- zqU-PbY Q ^ W%o XBQ - YBLi } PbY eq 4b@C B UC sebYc^ eq b4- zqU-PbY W%o |qBYL| = |qBXQ| = 90^> eqZb C f W%o WbX ^- szq ^%o yb <^- \ C^> C zqU-PbY W%o YBL - XBQ s eb@b4^ eb@- fCZ%o ai db@- <- @^S fS\ G C |QL| = |LB|> - zC@ |QB| = 2|LB|> eqZb C szq ^- BC UC 4b@ S L - Q qX @CC^- ^- zP cPj^- Wb @P sCW%o y-WC zqU-PbY W%o YBL - XBQ s eb@b4^ f eb\ Gc 1 : 2i yb <^- \ C^> C |XQ| = 2|LY| = 2vi



pbf^- Wb szq ^- AB UC <b^- @^S 4b@ SK - P qX @CC^- ^- zP cPj^- Wb @P sCW%o eqZb - U|KB| = 2|PB| = 2ai, Cb4C^> f b4C < eq f eb cZ b4s-P- zqU-PbY W UC (b \ vb)/2> W@Cb UC szq ^- - vb UC f W ^- @^ szq ^- i dYz zC@ =

$$S_{KBL} = \frac{2a \cdot v}{2} = a \cdot v$$

$$S_{PBQ} = \frac{a \cdot 2v}{2} = a \cdot v$$

, S@ \ G CSP b4s-P%o cPj^- Ui, ^- YLS\ W%o zb ~W-C C eq f CzW%o @fbIS\ C zqU-PbY Wbf > Wbq <P sebYc^ f q-PbY UC f q-PbY \ zfbq-PbY W ABCD> - zC@ s\ C @bW^- YS C b4s-P%o zfbq-PbY Wbf KLMN - PQRS s- cPj^- Ui

Komentár

, oC S^- qCC^> Wbq ^- \ eqS YS 4bYS seq f^C > cb ^- s fC \ SzC i f=, Wq-z b4C-s szC ^SCWbq szq zS| - { 4b@W%o- zb> C szC 4q YS- Wb s- \ bxCU bs zb> C zC f W%o s f ^CU Wb\ eb\ GcU dCz WUzb YPC> W@C UC zb UC@- < W cbf <P c-sz> UC zb ebzC4^ < @ fb^Si



V beq fb- SBqSW bf-W dCb Vbf<s
 ^- Wq USC qC C'S=BfW Vq UqSf-

Zadanie

] - <S'z bq^C zfbq Pcp4%ebYqW \ qC W%2019 x 2019i] - W @b\ < Pcp4bf ~fS@
 bWYS@ <S - Y> LGq4Cq> <Pq%o ^z \ ~ - Y4b \ - <P feq-fC U@^b < zbPbg zfbq<
 < Pcp4bf fC WszS2 x 2 f @%o-P - W @ q szY~i ? bW-zC> C^SCWC^ <S'z bq^C
 C'Sz~UC q @Pcp4bf 2019 x 1 - Y4b 1 x 2019> Wbq b4s-P~UCS4 @f @q-P%q szY^i

Rieszenie

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] - zb\ >Wbq WqZ UCWbq <b < @ ^S> ^- ^G ^C < ^C ^- Y^ ^- zb\ >Wbq
 WqZ%o b@sC4 q ^G VGy C\ qC W f<S'z bq^g\ C\ C ~4bfb ^Cbz-e >f C'W>
 cb @W-C CeqCq @W%eYz - Ueqsz e<C- ^- be-W
 VGy C f U@^b\ zfbq-S2 x 2 \ ~sS 4%o f C'W%oq szY%q ^<C> ^C\ ~ 4%o @f-
 cpf^ - W WqZ%o- s-sC@S <SP ebYqW Pi yb<^ - \ C^> C f U@^b\ q @Cfcsei sz e<Sg
 \ ~sS 4%o- seb @f q ^C @q-P%Wqzbf i, Ws f ^C U W\ q @CY^ @f q ^C WqZ%o
 z Ws\ CseYSSeb@ SC'W%o- @ ^Si, W'SC>z W C'Sz~UC q @f Wbq\ \ ~sS 4%o fS <
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 ~ Y^ U@^ - WqZS' - ff - fb\ @bYb\ ebYqW C>f eq fb\ @bYb\ Ag ? beY \ C
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 A>C - UD ~ s f-seb U@^b\ <2x2 zfbqbf>Wbq b4s-P-U zzb ebYqWb] -
 zzb ebYqWb zC@ @beY \ CWqZ B - bsz ^ ^- @f- zfbqC2x2>f Wbq <P ^-
 f W @b\ <P 4- ~ Y^ U@^ - WqZS' - ff - fb\ @bYb\ ebYqW A>f eq fb\ @bYb\

Cg yCq < fS@ \ G > C s \ C @bsz- Y qf^ - W q @bW- W eqf i Š - qfC sS f S ^ S \ G >
 C < zbPzb qxebY C^S f @%d@^ b ^ - q^C @beY \ C y- Y q @bW? bsz- ^ C C zC@
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 f ^ SW zqsz e < C > f Wbq < P s- be- WU Y @f W C %? bW- Y s \ C z C @ > C- W ^ C Q
 C f Sz- LC q @bWf Wbq \ s- szqC@ U S4 | @q- P % W C z f ~ - q S C C f Sz- LC z- W sz e C >
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Komentár

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 , z b \ z b e q e - @ C - Y 4 b Y z q C4 q b 4 q f S < \ b ^ bsz - > W ^ S < W ^ < b f > seq-f ^ C
 q C C ^ S - U z W f S C @ b W z b \ ~ > C C f Sz- LC - s e b q @ ^ S - W f b f b q f b \ q C C i

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 z C q V b f - < > [- q S [- s q ^ - > V q s z ^ - [S Y ^ b f - > ? - ^ S Y a ^ @ > Š - (- ^ - a ^ z W f S b f -

Zadania 2. série úloh letného semestra

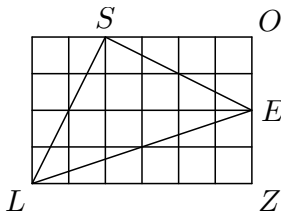
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YP- |

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zhf> z Ws- SP eqfC Cq^ fCW^ S b cEepWif i, Ws- WXbzzS\ ~ fCW eqf@- U
zC^? SzCqf> z Ws- eqfC Cq^ fCW^ S b y- Y <P D qWif i, W lC eqfC Cq^ fCW
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Yp J

VbqCYs <P<CY<4Y S sb sfbIS SeS- qW S- sfbIS S@ szU \ SebsY<P-ε\ S- zb
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\ bYsb\ sS C- W4%f-s 4bY f \ Sz^bszS@f- Wf z fS <- W lC zCq <- ebzb\ 4%
lC@b < f-s b@S Yb> z W4%f- ebCz 4bY@CzC^ ebCzb\ ~çC<hf> Wbq s zCq <
f \ Sz^bszS pbf^ Wb- W4%4bYebCz ~çC<hf f \ Sz^bszS@fbU-sb4^ - lC@C
< ^SP 4%ebzb\ b@S CY z W4%SP ebCz 4bY@CzC^ ebCzb\ @C> Wbq s zCq <
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fC- Wsbfg- f%Pb@ b- @ebfCzq - seb c- ^- UfS <I \ CzS ^sW@P @b bf feb-
sS%zP fS%q Oq s- Wf^ç eb fbeqC@ @Pb@~zb\ ebCzC - Pbf i? SzCq f%Rq>- WlC
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- á cc - Pb<P>
- á c| - Pb<Pi

Yp v

, P-lS q szC \ ^bPb szp\ bf - S' <P q szY^> Wbq ebWj f U zfbq< TRAPI | -
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	GSSe a YU	Šu	ŠVφJVB	D Q Q Q Q Q CE	cv
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] →bf= MATIK Vbc eb^@C^c^ \ -zC -zSW sC S-q
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 dqf@bfC@C<W-HWY> f4-f4 |>Cec IJ Vb SC
 Š@q C^SCrypa | >TGC^^-I>Cec IJ Vb SC

Organizačný poriadok korešpondenčných matematických seminárov Malynár, Matik,
 STROM je zaregistrovaný na Ministerstve školstva, vedy, výskumu a športu Slovenskej
 republiky pod číslom 2017/13750:2-10B0.