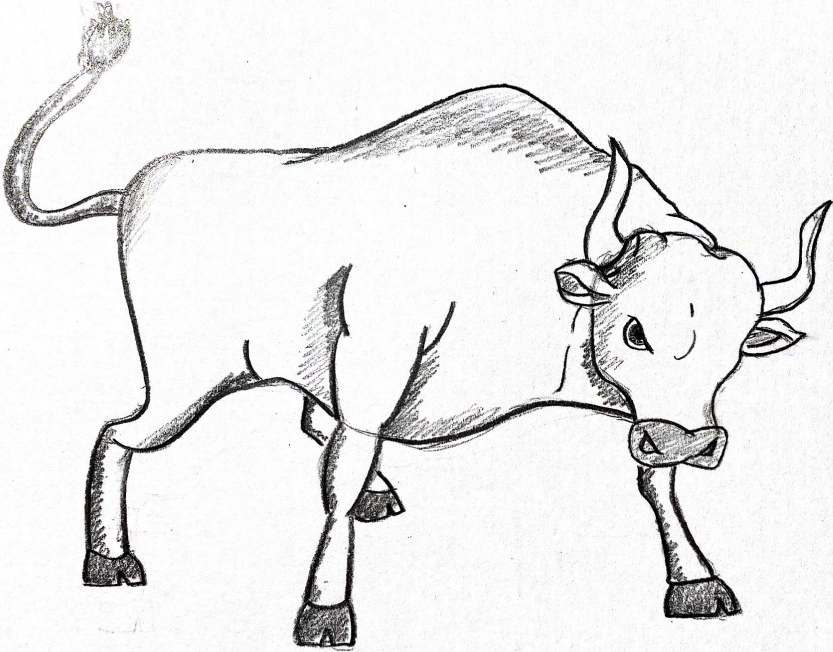


KOREŠPONDENČNÝ MATEMATICKÝ SEMINÁR

# MATIK

Číslo 5 – Ročník 34

[matik.strom.sk](http://matik.strom.sk)



# Ahoj!

yfbU ~ ebP - @ <Uf^C ^G ^SWb ÿ- YSC f% @ ^SC MATIK -> f Vbcp \ ^-U@C ^SOC  
ebq @C eb eqfCUs qSzbPzb sC Gszq >- YC - U^ - C f: bcpf qC C^ Si | G - 4 @ Uf - W  
Cs \ CC zCY^ f ebY- sC-z- W-qSC ^Ceb- f-U- e-szSs- @b @q-PCUs qG r q @bs b-  
bc- Wf \ CyfbUC ÿ- YSC qC C^S F

fC @ <SMATIK-

## 2% z daní

, UzC^zb qW \ ~ f- Sqb@S fC^bf |h <b sfbUSP @ ^ fCqU becpseC ^ \ bq- Q  
^S - S \ >- W s \ C \ %f@W^ <- ^SWbq - {h g  
dC^S <C < sW^ < |h f% f \ C ^- ebWpSC ç- szS^ - W @bf seblC^ <P s - WsSz- \ S  
eqC f-s fWe cpf- ^SC ç- sbeSbf> eb zbf^ > <C^%d^ - s - S <P>- WsSz%d^ - s szqC@C  
^S <P g  
; P<C C f-s eqCb ebedqs >- 4%szC q@Sb \ > çY^b \ sfbUCU 4Y WU- Uf< @S Y^CU  
q@S %s- sC@b \ - ebWUC- U~ @ \ ~yb \ ^- ~Y- Sebcp eq- f Yb^ - SP - WsSz- P  
- ebedqsSSSP>- 4%sfbl~ zq W~ eb@ebqSS^ - ~ @b4qfb^ <W çS^bs - eb\ bPY  
z \ sWeS^C \ Y @ <P <SC- fC@b \ <P ~ @ <- 4C eCç- U <SP <Pb@z <Pzb - s^ <P sCQ  
\ S^ - cpf> Wbq z W 4b ~UCG dbcp eq- f UC S \ > çb eqC f-s <^ - \ C^ - s szqC@C^SC  
çb f \ @ - f qC C^S YP sC S^ - q >- f%afC^S^S \ > Cz- Wb eb@ebqS - Uf - cp- fbU  
- eqseCU W \ %Y eY \ ~ zq- fC^S- f - Pb fb^ Pb ç- s-i  
dbzqC^ S^Hq \ <C b zb \ >- W @ cpf |h > ^- UCzC ^- szq^W ^ - Pb <@q- C^S  
Pzes=ww @q- <C^SG szqb\ i s WsWw @q- <C^SCW eCq <C^z- w- q @S f \ b@ebfSC C  
^ - ~4bfb^ bz- W@bP - @b \ ^- Ceb@ebq%U \ - SB \ ^- S^H2szqb\ i s W  
p- WUC F

## Tábor mladých matematikov

aeb 4%zb 4bY <- eq- @^S%4G y [ [ Wmyb fCq- ^CfS \ G ] b s \ CsSSz > CzC^zb  
qbWzb ~qSC <Szs ^G<P<C Cf dqb- UzC^zb qW-P%sz \ Ceq 4- @ <SP sC@ \ - Wbf  
S - 4- @ <SP @q-P-Wbf r y-4bq \ Y @ <P \ - zC - zMbf> Wbq s- 4- @C W^ -  
| {i Q| CE - ~L- sz- | Cfc f fd ? CzJS ^sW O~z F  
d z- s > çb UCzby [ [ mR@Cb D@ bf eb4%w^ - 4S <- ~U \ - f \ eqLq \ b \ > f WbQ  
q \ ^G \ SC <P 4 <- 4 f- \ > - zC - zSW - eqUC ^ - sebYç^bs i dbx f^W s eb@p4Q  
^ \ SS^Hq \ S \ S - eqPY- W ^- UC ^- ^- SP ..C4bf <P szq^W-Pi | G - 4- @^S  
f - W C We- <S- UC b4 \ C@ C^ -> eqzb s eqPYsC \ ^Cç- WU^ - ebsY@^ <P f - i  
yC \ Cs ^- yC4 F

## Vzorové riešenia 1. série úloh letného semestra



beq fbf Y= zCfb, - -W X-U- | Sbzbj-  
 $\wedge$ -UWq USC qC C^S=X-W- T-<W>a YfCqR C^-^

38 qC C^

### Zadanie

; P~Yb - , S^<C^>b s <^-\ S sfbU~ <-\Yz^b~ ebf-Pb-i , 4%& S^P C^%db ^Szb\  
 b@^SM^Pb ^C@b fC@C^S- f ^SCWbq @^SW @\ ~ W\ ><-zS qb <f%^\ @^S fq fS  
 eq f@-i; P~Yb W\ C f @%Y^ f szqC@->sb4bz~ - ^C@C~ - , S^<C^>b Pbfq eq f@-  
 Y^ f szqC@-> zfqzW- sb4bz-i V G^ s- szqZ^S f eqC@SS^Sd-Y<<b @S, qb4- Y^>  
 eq^C^PbYzC^>zb q^ Pbfq

á ; P~Yb=3~b^Lsq^bF, qCq sb\ W\ - Y

á , S^<C^>b=; S bF, UUF

, SzC U@^b ^-q^C~qS>f Wbq @C eq^C^PbYzC^>zb q^ Pbfqmr fbU- b@ebfGy <@ Q  
 fb@^SG

### Riešenie

R^Hq <S<b <-@^S sS<-e C C @b z-4- W%fd <^-\ C^>-> C f @^ @C Pbfq  
 @^ q^YfCWeq f@- V <^-\ C^>-> C f @^ @C W\ G^=

	eb^@CbW	~zbqW	szqC@	zfqzW	eS zbW	sb4bz-	^C@C-
; P~Yb	d	d	V	d	d	V	V
, S^<C^>b	V	V	d	d	V	d	V

Šb<-@^S fS\ G C; P~Yb-U, S^<C^>b zfq@S > C f qCq W\ - Y db q\ Cs->-W zb  
 H^L~IC< S^P ebP-@-i

[ C zC@ PbfqS eq f@- Y^4b W\ - i, W Pbfq\ eq f@->- zC@ f qCq sb\  
 ^-b- UW\ - Yz- W@C^ @C U d @C - f qCq U @C \ ~sCY4%V @C i, W W\ C >  
 - zC@ f qCq sb\ ^CW\ - Y- Y PbfqS\sb\ eq f@->-z W@C^ @C U V @C - f qCq U  
 @C \ ~sCY4%od @C i

db@- z-4- W%o - P^W - q\ G f Wbq @^S\ bP^S; P~Yb - , S^<C^>b ebfC@ sfbC  
 f qW%

; P~Yb=

á d @C >Wbq \ ~ eqC@P-@- V @C = zfqzW eb^@CbW

á V @C >Wbq \ ~ eqC@P-@- d @C = szqC@ >sb4bz-

,  $S < C < b =$


$$a \cdot d \cdot C > Wbq \setminus \sim eqC @ P @ - V @ C = szqC @ > sb4bz$$

$$a \cdot V @ C > Wbq \setminus \sim eqC @ P @ - d @ C = eS zbW ^ C @ C -$$

yCq < s - ebx q\ C ^ - zb > W @ % \ b P S b @ ^ S C f q W % b 4 b < P Y e < b f s q - s ^ G [ b P Y b  
zb z W 4 % f szqC @ - Y 4 b f sb4bz - i Š b < - @ ^ < P S H b q \ - S z C @ ^ C @ b W - C \ C W @ ^ b Q  
< ^ - q ^ C ~ q S > W @ % s - q P b f b q ~ @ S Y

**Komentár**

, ° q S - < f - s < f Y @ Y Y P ~ ^ - \_ 4 b @ b f i a e S ~ szC q C S S Y P ~ se sb4b \ > - W b f b f b Q  
q b f b \ q C C ^ > - Y 4 b szC q b 4 C q Y U @ ^ b z Y f @ ^ S b @ e b ^ C W - e b ^ C @ C ~ e b s z - e ^ C >  
q C C ^ S 4 b S e C W i , C - < f - s f - W - 4 - @ b ^ - f % q C Z C ^ S C H W - > C W q ^ S W b @ G s  
W \ G z W f C q \ \ ~ s Y P b f b q eq f @ - ^ - b e - W | S W b q < f - s z C ^ C @ b s z - z b C ^ C  
f % q C Z S S - e q C b e q - f C e b ^ C b W z f q b W s z q C @ - sb4bz - f % b b f - U ; P - Y b j S - szqC @ >  
sb4bz > e S z b W ^ C @ C - f % b b f - U , S ^ C < b f S a e s z b szC S 4 - e b j C @ Y S C z b f % Y f -  
< z - 4 - W % a z C ^ G - 4 @ U C e b b q ^ C q z - < - @ ^ S C Y P % f e z Y s \ C s - > q S f S \ C  
U @ ^ b ^ - q ^ C ~ q S @ C > f W b q q P b f b q e q C P b Y ^ S C > f W b q @ ^ S \ b P b Y e q C P ^ g  
g

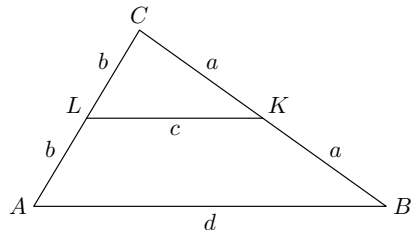
  $beq f b f - Y = [ - q S ^ , V b e \% V b e q - \Delta \% T - \Delta p S P ^ - f s W ]$  37 q C C ^  
^ - W q U S C q C C S = f C z W % a q b b f q C C S

**Zadanie**

Gq ^ < G s < @ b s z - Y b @ , S ^ C ^ - ^ - f q C S C - \ ~ Y z s z q b \ S z - U \ S e q C e - W \ S , \ ~ Q  
Y z \ - z f - q z q b U P b Y W A B C i a < ^ - q \ C s z q C @ % s z q ^ C B - C A e s \ C ^ - \ S K - L i  
d q C e - W s U @ b \ e q f e q f C ^ \ e q C 3 S ^ < - z f - q z f b q - P b Y W A B K L \ - b 4 f b @  
c C E \ - e q C e - W s H z W b ~ ; P - Y z f - q z q b U P b Y W K L C \ - b 4 f b @ v < i , W - U C  
@ W z q C z U z - U C e q C e - W % z f - q - s C W % K L m

**Riešenie**

3 b @ K U C szqC @ b \ s C S W % C B > < q b P b f % Q  
e Y f - > C s C W % C K - B K \ - U q f ^ - W  
@ W > b ^ - q \ C U a i d b @ b 4 ^ C 4 b @ L U C szqC Q  
@ b \ s C S W % C A > - z C @ s C S W % C L - A L \ - U  
q f ^ - W @ W > b ^ - q \ C U b i ? W s C S W % K L  
b ^ - q \ C C > @ W s C S W % A B b ^ - q \ C d i



Š b 4 q - W f % Y f - > C b 4 f b @ z f b q - P b Y W A B K L s - q f ^ - s q z ~ a + b + c + d  
- b 4 f b @ z q b U P b Y W L K C s - q f ^ - s q z ~ a + b + c i

a 4fb@ zfbq-PbY W ABKL UCbJ < \ j°ç - Wb b4fb@zpbPbY W LKC>eqf\b  
 UC@P Y^ b szq ^~ di Š zbPb f°eY f-> Cd = J < \ = jABji  
 , S\ G> C sCçW KL UCszqC@^b~ eqCçWb~ zpbPbY W ABC>eqzb C4b@%K - L  
 s szqC@%UCPb szq^ BC - ACi Š fYsz^bsz szqC@UCeqCçW%fS\ G> C@ W sCçW%  
 KL UCçf^~ebYfSS@ W%o sCçW%AB>- zC@ jKLj = | < \ i

**Komentár**

, °ç S^C< f-s s- eb@ çSb @beq <bf- s- Wseq-f^C\ ~ çC C^S>ç-szb s- f - Wsz- Yb>  
 CszC^→ ^Cj%çCZSSzb> CVX UCebYfSb~ < , 3 eqCb>Y4b UCszqC@^b~ eqCçWb~i  
 | SCUCzb z- W < U f^ >- Wb 4%ç- \ bPYb <@ >z- Wzb ^- 4-@ <C^G- 4-@^Sçseb\ C^  
 çg  
 a Wç\ zbPb s\ Cs- szqZSCzCs çC C^S\ S-Wbq eqC@ebWf@ Ys- CzpbPbY W 3;  
 UCçf^bq \ C^ ^ - Y4b çf^bszq ^^ >~ @ ^SC f - Wzbzb ^Czfq@Sbi rb <- @ ^ \ YP%  
 eqzb zç4- <- b4-P-@- fç\ Sbe-zq^C>- 4%szC bçq <~ ^CçC SŠ^Sçb S^ tg



beq fbf Ys-p 4Cqz r- 4bfç W , @CYObqf-zPbf-  
 ^- Wç USC çC C^S = pSP- q@dçWçqV-z-q^- a szçz-lbf-

31 çC C^

**Zadanie**

Š-Pq @ d-Y << b @S, çb4- Y^b UC çx @CC^—^ - @Cj° <-Pb^bf b@CC^ <P <Pb@^ çQ  
 W\ S- ~sebçf @ ^ <P @b zfbqç- 3 3i | SCWbq <-Pb^%ç ~-<- fqç eqç fçqUbs  
 f<-<^ - çC^ ^- eY^SW sSfb- Hq4b-g ? b W @ Pb <-Pb^~ f eY^SW s\ C fe s Yç sYb  
 bx ^- ç~U <C ebçCz ~-<- fqçz <P <-Pb^bf fsSf <P ebYçbWçs Wbq \ S@ ^ <-Pb^ s-Q  
 sC@>çS~ <Pb@^ çW\ - Y4b çPb\ i ] - b4q<W fS@zC eqWf @>- Wb \ C f%çq  
 eY^SW f Wbq\ UC s çCz ç sCYf <-Pb^b<P 16i Šszç> Wb Wb ÇsZ-UC q <^%çP \ b Q  
 ^bsz>- Wb ~-<- fqç <-Pb^%ç 4%ç\ C^ - eY^SW \ - Ys çCz ç sCY17 feççb\ ~-<- fqçS  
 <-Pb^bf>Wbq f^SWSzbçC^ \ UC@^ Pb ~-<- fqçS >q-z \ C<- zCSz g

2	1	2
3	2	2
1	2	1

Riešenie

$] - eY^{\wedge}SW \setminus \rightarrow C \{ q \wedge C @ q - P \% e b Y \dot{c} b W f \leftarrow P b \wedge b f g$

á c szq@bf ebYcWb>Wbq s-s@ s s\ S\ SebycW\ S- z@ U@Pb ~- fcz \ s ^\rightarrow W<CWF \ ~ s cz~ eqfbcz D

á J qbPbf ebYcW>Wbq s-s@S s zq\ SebycW\ S-z@ SP ~- fcz \ eqfbcz Q z \ C \ W<CWF \ ~ s cz~

á J ebYcW f szq@b<P szq^>Wbq s-s@S s eS z\ SebycW\ S-z@ SP ~- fcz \ eqfbcz \ CI W<CWF \ ~ s cz~

, S\ Cz@ > CW<CWF \ ~ s cz~ fS\ Ceqf@f Y^ çsY {>I - D>eb\ b<b- Wbq <P ebzC4-U C < sW s çz cui S-qbfc eYz> C { - I \ C C < - qz \ - †S -YC JQVz- D^- U S < q < i

æsYb cu Ueqfbc sY>z WCpb eb- S\ S4 qb^ - W < P @q-Pbf ebYcbW^C@bsz ^\ C feqzb Ccu ^SUC@S^ zq\ S-eS z\ S- ^S s\ S Sg

VGy Ccu ^SUC@S^ zq\ S fS\ C C U Y^ < ebYcbWs ebqzb \ { < -Pb^%δCQ ebsWf @> G eqzb \ ~s\ Ceb- S -seb U@^b < ebYcbWs ebqzb \ I - Y4b Di æsY

I>D- cu \ - U eb @CC^ zq\ Sqf^ - W < f%bW- zb | i æsY \ \ -eb @CC^ zq\ S < f%bW Ezb < ^ \ C^ → C^ G P Pbeqfz \ CW W \ ~W fCWç sY > < f%bWzbPz s cz-

eb @CC^ zq\ S ^G \ C^ i , W4%δ C ~- fçCS eqfC | ebYcW s ebqzb \ I - Y4b D fçS Cs çz z < Pzb ebYcbW4%4bYcEç - Y4b cvg>z- W4%4bY: f%bW<CWF Pbs cz-

eb @CC^ zq\ Scf- eqzb 4%δ C^ \ bPS@bsz- <CWF s çz cu>z^ @f < f%bW | g , W4%δ C z- Wzb ebYcW ~- fçCS { fçS Cs çz z < Pzb ebYcbW4%4bYcI > cD > | c

- Y4b | Jg>z- W4%4bY: f%bWs cz- eb @CC^ zq\ SCE- eqzb 4%δ C^ \ bPS@bsz- <CWF s çz cu>z^ @f < f%bW | g , W4%δ C z- Wzb ebYcW ~- fçCS -seb J>

z- W4%4bY^ - U C^ \ b ^ s çz 4 5 = 20 > çb U fS < - Wb cui S zbPzb < Sz \ C < P < C C ~- fçC eqfCU@^b < ebYcbW Wbq Ws cz- eqf U I - Y4b Di

VGy C fS\ C \ ~s\ C ~- fçC eqfCU@^b < ebYcbW Wbq eqf U I - D> z- W ^\rightarrow < zbPb f%Y f U | \ b ^bsz-

á , W<- fçC Cc szq@bf ebYcWb>z- W → bsz ^Cs çz > Wbq @beY \ Czq\ - qbPbf \ SebycW\ S

á , W<- fçC Cc ebYcWb f szq@C szq ^%bsz ^C ^\rightarrow s çz c | > Wbq < sW \ C ~- fçz \ f CzW < P J qbPbf < P ebYcbW

db@sz- z^ UC zCebfC@ > C P b - S f b4b < P eqe- @b < P fS\ C < fbY çx Yç^ ebYcW f \rightarrow C ^ - eqWf @ J ebYcW > Wbq s f szq@b < P szq^ > z- Ws- f CzWqz U Y^

< - U@^ ~ \ b ^bs > eqzb C fC@S f ^SW bzbC^ \ U@^ Pb ~- fçz Si y \ zb s \ C z@ ~W- Y C \ → Ceqs^C @fC qC C^ S > Wbq s ebe s ^ f%Si

**Komentár**

y- W Gq f CzSszC ^- Ysb4C seqf^C \ b ^bszS- cb ^-s s- \ b cU C fC \ SzC i | ] - U  
ç- szCU \ fz- W Gq U@S \ gep4Y \ b \ f - W4bY zb> CszC ^G-W- Y- C \ b ^bszS-  
Vbq szC ^- Ys sWzbç^C zC U@S seqf^G , zUzb YPC zb 4bY s < C bçSfS^ >  
- eqCb szC \ bPYS \ - eb<S> C - U 4%abç^ > ^b zb 4bP~ S ^SC UeYz^ - qL- \ Cz  
=? i | ] - 4-@ < CeqCb \ %YS C ^- zb> Cs ç- s b- qC C^S U- U-W- ^SC zbPb> Cz U  
zb U@S seqf^G

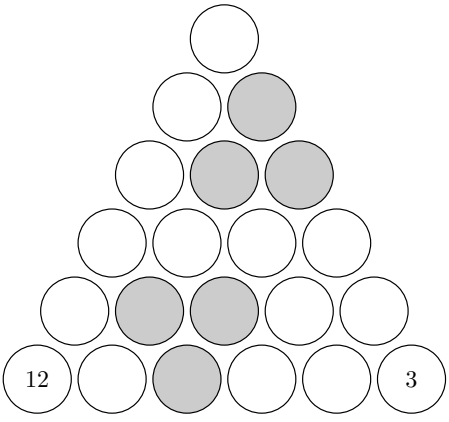


beq jbf Y? - ^b a ^@ - 3S K~qSW-  
^- Uq USC qC C^S = [ - qS - a s-sW>a ^@qUVq-YW

| u qC C^

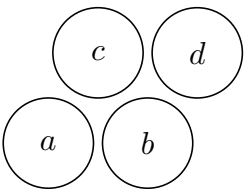
**Zadanie**

a 4q-s>@b Vbq Pb Gq ^<G<- f%of>\ -zf-qzqU-PbY W < Y C^ Pb < Vq-Pbf - Wb ^-  
b4q+Wi Gq ^<G<- <P<C f%S @b W @ Pb Vq-P~ zb Vb WSCZVf>- 4%4bYebçZ WSCZQ  
Vbf f W @UzqU-SVq-Pbf>Vbq zfbçS \ -Y zqU-PbY Wf^ - b4q+W fs@zC f%dlq  
4C^ eqW @%@fb<P z W<P zqU-PbY Vbfg @CSC^ eS zS S ? b Vq-Pbf f @bY <P  
qPb<P Ulf - Vq^b b@ < YszS3S ^<- f%SY 12 WSCZVf - 3 WSCZWq Wq < ^CebçZ%  
WSCZVf \ C Gq ^<G<- f%S @b ^- UfqP^CUSPb Vq-P~m] -Usc f CzWda b ^bszS

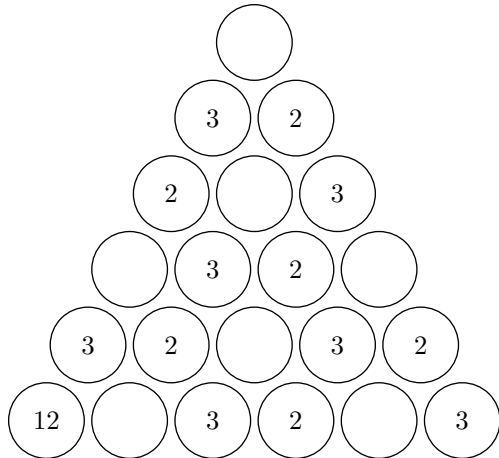


**Riešenie**

, Wsf%4CqC C z%qs-s@S <Wq-P%feb | < U@^ Pb q @-g-  
f%zfbçS | zqU-PbY W%oacb - cbd> - Wb ^- b4q+Wi ySz  
zqU-PbY Wda - U sebYç^ Vq-P%b>c i yb < ^- \ C^ -> C- 4%  
b4- zCzb zqU-PbY Wda - Ys çZ WSCZVf @CSC^ I>Vq-P%  
ad \ - U qf^ - W < f%bWeb @CC^ Ii yzb eYz > eqCb C  
f b4b<P zqU-PbY Wb<P 4-@Cs çZ < f%Vb Vq-Pbf c>b>zC@  
ebzq4-U Ceqqz - qf^ - W < f%bW- 4%S \ C @bsz YçsYb @CSC^ eS zS S



dbx fS C s ^- z Wzb J Vq-P%gf - fb\ @bYb\ qpP-i, zCb zbqfSUC f-fb @bY  
 Vq-P s c| WYSCZ\ S Š zbPb fS\ C Vq-P feq fb Pbq 4-@C\ - qf^-W <f%bW  
 eb @CC^ eS zS S-Wb c|>çb UC |i yCq< \ C C ^-tš ÿ-Ys- zfbqf~> f VbqU  
 s- f%bW%-UC zC^zb Vq-P sb<f%bW\ |>fÿ-W çb\ ~ ^-UC Cÿ-Y Vq-P sb<f%bW\  
 |i y \ zb se sb4b\ fS\ C ^-tš f CZW%Vq-P%bWbq ^-@ U <f%bW|i  
 pbf^-W \ Cz @- eb- UC C-Ueq f zfbqfS f eq fb\ @bYb\ qpP-> VqC ebx ^-C  
 Vq-P sb<f%bW\ {i a e° fS\ C qf^-W\ ebsz-eb\ ^-tš f CZW%Vq-P%bWbq ^-@  
 @ U <f%bW|i



, Wb fS^b< b4q+W>eb @beYC^ bsz-Yf W @b\ zqU-PbY W eq-fCU@C^ eq-@%  
 Vq-Pi pbf^-W UCsz-YCeq-@%U^-Ufq-P^CU Vq-Pi r çCz<f%bWf f W @b\ zqU  
 ~PbY W UC<-zS I fY4b |j { E Ig yb<^- \ C^> C @beq-@%P Vq-Pbf f zqU  
 ~PbY W<P \ ~s s YC^ z-WçsY>Wbq UCzC ^-sb4W\ I>-4%4bYs çCz @CS^  
 eS zS S yCq< s\ C~ f%bYSS-U^-Ufq-P^CU Vq-P- <SzSS\ C> Cz \ \~s 4%  
 ebçZ WYSCZWf>Wbq UC@CS^ eS zS S

**Komentár**

y- W Cq f Cz-SszC^- Yseq-f%gf sY@bWçb ^-s zC i, WszC\ - Yeb@b4^ qC C^SC  
 -Wb f ç-sbeSC^ -Uç-szCUC szC 4b@%szq zSS fzC@%-WszC ^C@bsz-zbç^C f%bYfCzSS  
 eqçb Vq-P%beqzSsC4C f zfbq-PbY W \ -U qf^-W <f%bW\ SCWbq < f-s sWQ  
 -Y- Uf CZW%W\ 4S^-<C<f%bWf f zqU-PbY W f-fb @bY, z-Wb\ eqe-@C f-W  
 \ ~s zC f%bW - ^-bx-Uf CZW%b ^bsz>fcb 4%z~ ^C\ -Y4%eqb4Y\ >Wç C SP UC  
 YC^ e° tgg





beq fb f Y r q K e q f y S W r Y s b f  
 $\wedge$  - W q USC q C C S = | - q W O b q f - z P > V - z W G q 4 - Y b f -

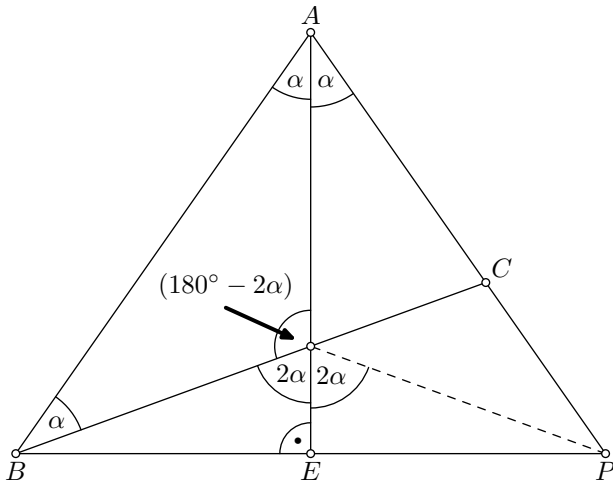
| u q C C

**Zadanie**

3 S ^ S - \ - \ - Š S z U ~ sz q - eb z q 4 - U s W @ q - b e q U s @ b d - q b R ^ - \ b q  
 q z b i T U 4 C ^ - z q s - \ - z f q z p U P b Y W A B C > f W b q p \ ~ P b Y C A B U @ f W q z  
 f o c - W ~ P b Y A B C - < - q f C ~ P b Y A B C U \ C ^ - W 4 5 sz - e b f i a s ~ P Y C A B  
 e q z ^ - sz q ^ ~ B C f 4 b @ C D i 3 b @ E Y C ^ - e q \ W A D > - Y \ S b z p U P b Y W  
 A B C z W C f C W s ~ P Y B E A U \_ C e s z - e b f i V G y e q @ S C s C W % A C - B E  
 ^ - e q \ W % z - W s - e q z ^ f 4 b @ C P i V b W W q z f o c U ~ P b Y B D P - W ~ P b Y A B C m

**Riešenie**

| - U W q s b x ^ - c \ C f C W s ~ P Y A B C - W i V G y C U C ~ P b Y B A C @ f - W q z f o c  
 - W ~ P b Y A B C - U P b s > e q \ W A D > @ C Y z C z b ~ P b Y ^ - @ f C P b @ ^ c - s z S 4 - @ C  
 e Y z > C j < B A D j = j < C A D j = i d b @ - ~ P b f B A D - A B D f z p U P b Y W A B D  
 f S \ C @ b e b c z - j < A D B j = 180 = 180 2 - ^ - s Y @ ^ C - U U P b s - s C @ ^  
 ~ P b Y j < E D B j = 2 i



y q U P b Y W % A B E - A P E \ - U < P b @ ^ sz q ^ ~ A E > - P b Y e q f q P b Y C A f g - U ~ P b Y  
 e q f q P b Y C E f 9 0 Q W G y C ~ P b Y B E A U e q f - 4 b @ P Y C ^ - e b Y e q \ W C B E G  
 Š z b P b f % Y f - > C z p U P b Y W % A B E - A P E s < P b @ ^ e b @ - f C % u s u i s C W P E  
 U e q z b < P b @ ^ - s B E i d C z p U P b Y W % D B E - D P E z C @ e Y z > C j P E j = j B E j >  
 j < B E D j = j < P E D j - \ - U < P b @ ^ sz q ^ ~ E D i Š z b P b f % Y f - > C z p U P b Y W %  
 D B E - D P E s < P b @ ^ e b @ - f C % s u s i  
 æ S C ~ P b Y B D E U q f ^ - W - W ~ P b Y P D E > z C @ 2 i Š Y C ^ \ z < P z ~ P b Y f ^ ^ S W C  
 ~ P b Y B D P > W q 4 - @ C f C W 4 i y C @ ~ P b Y B D P U C J Q W z f o c - W ~ P b Y A B C i

**Komentár**

y- W Cq f CZW > Vbq YP~ bbfk @ Ys- eb @ qb ^-ls seq f^C qC C^Si , °ç S-  
 < f-s ^- zb SY <G <Pb@bs zpbUPbY Vbf- Y4b Y^ @bebz z f Y ~P%qd - Ys- f - W  
 - Uz W> Vbq qC SSeqW @eb\ b-b- zpbUPbY W s W^Wj z^%oS-PY\ S  
 y- Wzb qC C^S \ ~s \ CbPb^bz \ C^ \ ebczb\ 4b@bf> Wç Czb ^C@bW<~U eq  
 f CZW zpbUPbY W%fs ^G ^- \ % ~PY\ g- YC^ eqCe-q W^Wj z^G f- \ Sf%4q ^ >  
 zpbUPbY W% zC@%qC C^S^C@bW<~U> C4-@Cf @%eYZS>- eqZb-s zb\ ~sWszC  
 f%qf qf- Q^- eqfWzb\ ~> C f-P%eb\ b-b- W^Wj z^%P ~PYf f-s \ ~ ^- fSç  
 ^- seq f^~ \ %çC^W ç



**beq fbf YV ~4b K C^çS**

^- Wq USC qC C^S=, Y^W 3-Yzb f->, Y@ qrY^S-

cD qC C^

**Zadanie**

; P-Yb - , S^C^<b s- ebP @ Ys- qPb@Ys- sfbUsebq f%qC S s 4bU\ \ %Yi Oq U  
 zC@ Pq> W@Cb@ \ - \ %šS%@bsz ^ b4q<bW s n 3 4b@ S- k sCqW\ Ss W^<bQ  
 f \ S4b@ Sf z <Pzb 4b@-Pi | - lçqf; P-Yb f%4CqC@f- < z <Pzb 4b@bf- b^ ^-ç SP  
 A - B - ebY W\ S^bW^ 4b@Ai dbzb\ Pq <-ç ^- -Pb\ , S^C^<b U Pb - P  
 f%çq-z W Ceb- ^SCW\ C eb ^CUWU sCqW @ U@q-P Pb W^<bf Pb 4b@-i | - Q  
 sC@~U; P-Ybf -P <\ - CU@~< sCqW b4q<Wi , z <Pzb -Pb-P s- ebsz-e^C  
 szqC@ Ui , Ws- W\ S^bWeb ^CUW\ ebçzC -Pbf eqçs- ^SC @ 4b@- B>z Wf%Bq-  
 , S^C^<b - ^- be- Wf%Bq-; P-Ybi dç<- @ ^CU Pb^bz C n>- W U^ - U^ç SC \ b ^ k  
 z- W> C; P-Yb fSC f @%f%Bq m

**Riešenie**

VGy CP - @ \ C ^- U^ç SC \ b ^ k>eqfWbq\ fSC f%Bq ; P-Yb>ebç q Cs ^- Pq>  
 W@Cs seblC sCqW~ W @ @f 4b@%a z W\ ege-@C; P-Yb <fbY 4b@%A - B  
 - ebY W\ S^bW- 4b@Ai VGy Cs seblC ^- f<-U\ f CZW 4b@%z Ws seblC  
 -UA - B>- zC@ , S^C^<b f eqfb\ -P~ \ Ceb- ^ W\ S^bW 4b@- A @ 4b@-  
 B>fj- W çb\ ~ f%Bq+  
 aeb - W4%o^ - b4q<W <P 4-Y S4- c sCqWm, z W\ ege-@C; P-Yb <fbY 4b@%  
 \ @ SWbq \ S sCqW ^CfC@çS- W 4b@%A - B fS^-ç zb @be- @C- W f PqC f%oSg  
 , S^C^<b ebzb\ \ ~s ebs- ^ W\ S^bW- ^CUW S^ 4b@yCzb 4b@~ U seblC  
 s 4b@B >- f - Wz zb sCqW; P-Yb \ Cf<-eoz <\ -<- iy \ e-@b s , S^C^<b  
 4-@C \ ~sC ebs- ^ ^- j-Y 4b@yzb \ ~ be- Wbf @bfzC@%W\ @ 4b@- B  
 ^C4-@C fSç S @- sCqWi , W, S^C^<b f eqC4CP- Pq%eb- ^SCW\ S^bWeb 4b@->  
 < Vbq Pb ~ ^CfC@ç sCqW @ 4b@- B>; P-Yb \ C b@szq^S ~4bfb ^ sCqW  
 fC@ <- @ 4b@- Bi , S^C^<b <U f^C f z Wzb Pq ^C \ C f%Bq >eqzb C eq@C  
 b f CZW% sCqW fC@ <C @ 4b@- B eq@z \ >^C s- z- \ @bsz ^G

, S\ Cz@ > Ck UCpf^ \ - †S -Y\ ~ ebçz~ sÇçSW ^~sci, W UCf - W - †S -Y  
 ^%ebçz sÇçSWš W @ Pb 4b@~ f\ → C S P ng fC@C sÇçW @b bsz-z^ <P n 1  
 4b@bfi ] G\ S\ C f - W - 4 @ > C sÇçW \ -@f Vb^ <C- \ %t ebzç4-U\ C<- ebç Q  
 z- S4- q <i | - †S -Y%ebçz sÇçSWCeqçz  $\frac{n(n-1)}{2}$ ; Š zbPb f%Y f-> C \ - †S -Y-  
 Pb@^bz- k UC  $\frac{n(n-1)}{2}$  li

**Komentár**

] - Uç- szCub~ <P%4b~ 4bYb zb> CszC~f bf Y> C, S<C^<b f @%ebs~^SCW\ SC^bW  
 ^- 4b@< Vbq Pb fC@C sÇçW @b 4b@~ Bi TC zb s <C S z-Sz f^C> ^b zb-zb f-Pb-  
 eqç@ebW @-zC C, S<C^<b 4-@C Pq eb@- ^CJWU szq z LSG VGy C f - WP - @-zC  
 f - <^ szq z LS eqç; P~Y >z-\ ~s H^Lbf ^C-P, S<C^<b Pq- VbVb fCWF- UVGy  
 4%f @%oç4SY eYC^ -Pb@^ -Pg dççz 4bYb ^~z^ seb\ C^ -seb zb> C f z-Vb\  
 eqe-@C\ C; P~Yb <\ -<- eYC ~4bfb ^ sÇçWi ybzb UC @ fb@Vf YVbq \ ~  
 f^ç S- < f-s eqçY b 4b@

, ~zbçf f<bçpf <P çç C^ =BçW3Cçz->, SW çç 3ççS^bf->[ -ççS, YCçz K4 çç  
 d-zçWd- bçç Wp 4Cçz r-4bçç W -^Cz- rC\ -^SS^bf->yS G r< Ysbf-

**Zadania 2. série úloh letného semestra**

pS C^S eb YC^- UCWq@b {i \ -U | Cc

**Yp- c**

dq@^@f^b\ sSczCzq U 4q zS ; P- Yb>, S<C^<b- KShf ^^SybqCYB^Sjf<Sqx @CQ  
 bf Ybq@S^ eC^S<Cf eb\ GqC4 : 5 : 6>^- U C^U@bsz-f Y\ Y@ ; P- Yb- ^- Ujs <  
 ^- lz-q KShf ^^S dbzb\ > W^KShf ^^Pb^- Y\ zfCPb fb fYsz^CUM\ ^- zC; P-Q  
 Yb ^- fqbY C sKShf ^^Pb q-s qx @CS s, S<C^<b\ ^- ebYfSG, S<C^<b f- W  
 zqf Y^- zb\ >- 4% s- Uz zb q-s qx @CS^Gqf^b\ GqC- W<f^% U^C-z@ f eb\ Gq  
 4 : 5i, S<C^<b 4% abzS eb@- ; P- Yb/Pb ^- fP- @bsz- Yb 4000 Yq\ C^U- W eb@-  
 sfbUpi Vb W eC - < \ -@W e% q@S- ybqCYB^Sjf-bfm

**Yp- |**

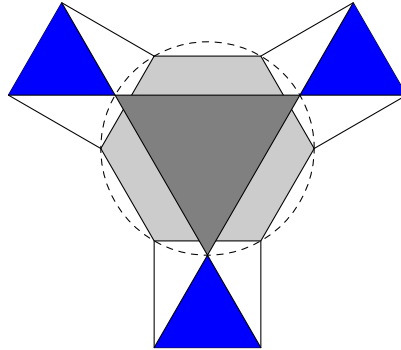
] - \ - Wq^ 4-%of d-Y<<b @S, qb4 Y^b UCW @ qbWeb.f ^ <P 21 Pbszi ] -  
 ebsY@^ <P @fb-P 4-Y-P s <- W @ \ qx @CS@zbzq-P s-Yeb s@C\ Pbszi } W-zC  
 C^SVMq zq U PbsS 4bY^- b4b-P 4-Y-P sebY f UC^Us-Yi

**Yp- {**

3S ^< eqSY ^- zb> CU , S<C^<b eb@f@- - z Ws- s ^ \ ebP-@ Y> CYz- Yz-Q  
 ^Sg Vb^Wj z^C Yz- YUC^ z ^Sgzf q- 101 QPbY Wi ] - lqf Pb@SY z ^Sqb<C  
 3S ^< - z \ Pb qx @CSY eb@ ~4bfb ^CU-PYeqCqW%3S ^< - U, S<C^<b fC@S  
 f @%Pb@S z ^Sgz- W- 4% s- qx @CSYeb@ ^S. S. fbY^CU-PYeqCqW% ^- @f \ ^bQ  
 Pb-PbY W%q -sY^C, S<C^<b<b4q Y<b<C CUC^< < qfCebf zf q- \ ^bPb-PbY W  
 - <^bf- Pb Pb@b\ b<C qx zqC zSY^- @f \ ^bPb-PbY Wj f qfC%eb@ ^ \ <fbQ  
 Y^CU-PYeqCqW%y- Wb szq@ fb P-@~ qfC%b<C >eqfb\ P-@W eqPq-zC^>Wb  
 ~ ^C C qx 4S S @C^ qfCei Vz< ^SP \ -f^dqf U<- szq z LS- - - Wmr fbu  
 b@ebfCy < @ fb^Sg

**Yp- J**

] - b4q-W fS@zCeY^ d-Y<<b @S, qb4 Y^b< Y C^ < eq fS@CY <P \ ^bPb-PbY Q  
 Wjfi r-Y ^- eq<C zf-q Gs ~PbY W- UC^Y ^- eqfb\ ebs-Pb@ zf-q z\ - fbsQ  
 f Pb zqP-PbY Ws fe s ^ @bzUSzCUM^ ^Sg V- @- zqP-P qf^<- W-P W\ ^-z  
 zf-q \ b@q Pb zqP-PbY W \ -b4s P 17i } qzCb4s P UC^Yg



**YP I**

d- YsS@b < b S- ^- ^bc^b\ szbYW eb W @UCeS @C<-eS-UC> Wb WbWqz~ Cq ^Q  
 <G<- eb@fSC@Y ; P-Yi dqf @fCçsY> Wbq ^-e-s-Ys 1 - li V- @ ^-sY@-U<C  
 çsY> Wbq sS<-e-s-Y4bYs çS^b\ eq@P@-U-SP@fbP<f C^ \ bciyb<^<-Q  
 \ C^> CçsY> çb sS^<-e-s-YebzçCUCeS @CIC1 1 + 1 = 2i } W-zC CçsY> çb  
 sS^<-e-s-Yeb2020CUCeS @G^SCIC@CSC^ çsY^ 4i

**YP v**

, b" ^-Y | {zUs çCs- , S<C^<b @x fC@> C3S^<- Pheb@fSC@Y - çx Pb@bYs <- Q  
 fq @S U> çf^<- Wb - Wb ebsY zCbz seqf%00 ; P-Yi | -P--- U s- zC@ eb b4fb@C  
 Wq-PbfUcS-Y@eçfb\ , S<C^<b b4CP^C<CY s-Y <- 360 sCW^@ 3S^<- 4C @f Q  
 Wq-z q <PYUCS- Wb, S<C^<b - ; P-Yb 4C zçWqz q <PYUCS- Wb, S<C^<bi Š-ç^<- U  
 < çf^<- WPb \ Sçz - 4CP-U>ebW\ s <^bf~ f Cç-Szq U ^GzçZ^ ^- zb\ Szb\  
 \ SçzG, W @P @b4- @bW%00- zC^zb çs- 4bYzçpU PbY W, S<C^<bçS^<- Q P-Yb  
 bszçp-PY m

*Poradie po 1. sérii letného semestra*

dbq @SC	[ C^b - eqC fSvW	pbc^ W	WbY	ci   i {i Ji Ii vi ; r
ci Qvi	X<-S ; PY@^—	Š_	K, [ æ, 3,	- - - - - IJ
	pSP- q@dçWq	Šu	KT, p  da	- - - - - Q IJ
	[ - çS^ a s-sW	Šu	Š? çf?   ,	- - - - - D IJ
	a YfçrC^ - ^	ŠD	K, YUVB	- - - - - Q IJ
	Bf- Vq ççf—	ŠD	K, YUVB	- - - u - - - IJ
	[ - çS^ Gq ^CW	ŠD	KsfGOŠ,	- - - - u - - - IJ
ui	yb\ — V~4çS-W	Š_	ŠVçfJVB	D - - - - - I{
Di Qcci	, Y@S qrY^S-	Š_	ŠVçfJVB	- - - u - - - I
	[ - z XS4-W	Š_	K, YUVB	D - - - - - D I
	pSP- q@, bçS-W	Š_	K, YUVB	D - - - - - D I
	[ SP-Y, bçS-W	Šu	K, YUVB	u D - - - - D I

dbq @cC	[ C^b - ecC fSvB	p bc^ W	VbY	ci   i { i Ji Ii vi ; r
c i Qc(i	a ^@qUjy zP	Šu	K, -q Š,	- - - D Q u Ic
	a ^@qUVq-ŠV	Š_	K, YUVB	- - u - - D Ic
cJi	, Y^W 3-Š-zbf-	Šu	; ŠpŠ-Š,	D J u D - - ICE
cli Qcvi	[ -qCWO bqf-zP	Š_	KVb^ da	u - u - - D J_
	, Cq^ŠW , b@S-Vbf-	Š_	K, YUVB	- - - I - D J_
cui	] -z-Š yWēbf-	ŠD	ŠXCfbr]	- u u { - u Jv
cDi Q CE	V-z-q^- G- qf~Vbf-	Š_	K, YUVB	- - - - - Q JI
	d-~Y^ yWēbf-	Š_	ŠXCfbr]	- - u { - D JI
	T-^W } qf~Vbf-	Šu	K, YUVB	- v v v - v JI
ci Q  i	X-W- T- <Vb	Š_	ŠVqJVB	- - u - - Q J{
	r b - K qf~Vbf-	Šu	ŠX  b fVb	- - u Q - Q J{
{i	[ - †S - 3C@- qf Vbf-	Š_	K, YUVB	D - - I - Q JCE
Ji	V-z-q^- a szCq-Lbf-	ŠD	; B] , ? ,	- - - - - Q Q {v
Ii	r-q P V Yēszb <W	Šu	e  ] ? -K	- { v   { Q Q
vi	[ - qzŠ ? ~@UW	Š_	r[ Y@d d	- - Q v v Q {CE
ui	T-^W b-Pbf-	Šu	Ka e- zVB	D D     Q Q  D
Di Q CE	] -z-Š dbŠ cS Vbf-	Š_	ŠVqJVB	- - - Q Q Q  u
	B\ \ - rz-U^cbf-	Šu	ŠdCq b] ,	- - Q Q Q Q  u
	Š- <- ^- 3C b f-	Šu	ŠGVq-Š;	CE - Q Q - Q  u
{ci Q {i	-4b\ q C^S Vbf-	Š_	KXŠ- ^%o	u - CE v Q J  v
	3bqS V q	Šu	K, YUVB	D - Q Q Q Q  v
	O- ^- r- 4bf-	Šu	i Š	v v u Q Q Q  v
{Ji	V- Ysz rC- ^ <b f-	Š_	Šr] dcOB	- c u Q u Q  J
{Ii	p- @bf ^ [ SŠ^Δ	ŠD	ŠVqJVB	- v J Q { Q
{vi	XC^W dCqf- ~Vbf-	Šu	ŠdCq b] ,	I   Q Q   Q cJ
			K%o i Ti	
{ui	yCqC- re-szbf-	Š_	yq ^bfswPb	J Q Q Q Q Q J
{Di	y- zS ^- [ S Cq- Vbf-	Šu	ŠdCq b] ,	CE c Q Q Q Q

